Closed-Form Expressions for the Parameters of Finned and Ridged Waveguides

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***Abstract*—Novel closed-form expressions for the cutoff frequency and the characteristic impedance of finned and ridged waveguides are presented. Agreement with previously published numerical data is better than one percent for all parameters of practical interest. The expressions considerably facilitate computer aided design and tolerance analysis of ridged waveguide structures without compromise in accuracy.**

I. Introduction

RIDGED WAVEGUIDES find many applications by virtue of their large inherent bandwidth and low characteristic impedance. Furthermore, planar microwave and millimeter-wave circuits of the type described by Konishi [1], [2] as well as fin lines [3], [4] can be analyzed and designed using ridged waveguide theory.

The ridged waveguide is well documented [5]–[9]. However, the designer must rely either on tabulated results or on design diagrams and graphs which are necessarily restricted to a limited selection of cross-sectional dimensions. If a different geometry is needed, one must either solve a transcendental equation (Transverse Resonance Method) or use a numerical technique. All these approaches lack the flexibility and speed desirable for routine design, especially in cases where the dimensions of the guide are continuously changing (as in tapers and matching sections). Furthermore, a tolerance analysis cannot be carried out easily with these methods.

In this paper, original closed-form expressions for the cutoff frequency and the characteristic impedance of the dominant mode in double- and single-ridged waveguide are derived. The special case of waveguides with thin ridges (finned waveguides) is treated first. Then, the more general case of ridges with finite thickness is considered. Expressions are based on perturbation theory and contain empirical correction terms to assure agreement better than ± one percent with various numerical techniques.

II. Analysis of Double-Ridged Waveguide by Perturbation Theory

A. Cutoff Frequency

It is well known that ridges in the E-plane of a waveguide lower the cutoff frequency of the dominant mode through capacitive loading. If the ridges are short and thin,

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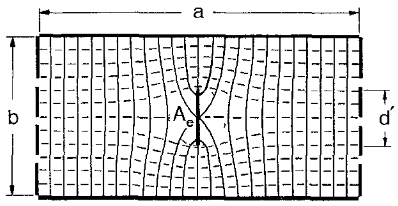
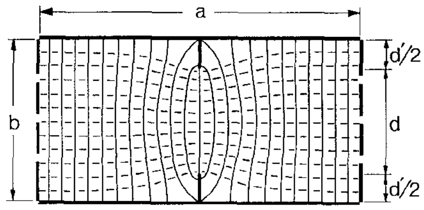


(a)



(b)

Fig. 1. (a) A conducting band perturbs the uniform electrostatic field between long parallel strips. The capacitance of the strip line is increased. (b) The reciprocal structure of Fig. 1(a) having a similarity to ridged waveguide. The increase in capacitance is the same, provided that $d = b - d'$.

their effect can be accurately evaluated through perturbation theory.

1) Waveguides with Short Fins: Fig. 1(a) shows the cross section of a region bounded by parallel conducting strips on top and bottom, and by magnetic walls on each side. The originally uniform electric field is perturbed by a thin conducting band suspended in the center. According to Wheeler [11], the relative increase in static capacitance of the line due to the band can be expressed as a ratio of effective areas

$$\frac{\Delta C_0}{C_0} = \frac{A_e}{A} \qquad (d' \ll b, a) \tag{1}$$

where $A_e = d'^2\pi/4$ is the effective area (circumscribed circle) of the band. $A = ab$ is the cross section of the line, and C_0 is its static capacitance before introduction of the band.

Exactly the same expression applies to the structure shown in Fig. 1(b), which is the reciprocal of Fig. 1(a).

Now consider a double-ridged waveguide formed by replacing the magnetic walls in Fig. 1(b) with electric walls. This imposes a sinusoidal field distribution. At cutoff, the equivalent line capacitance is proportional to the stored field energy and is half the static value

$$C_1 = \frac{1}{2} C_0.$$

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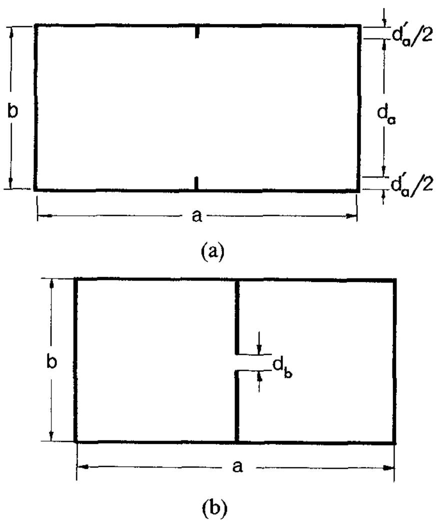

Fig. 2. (a) Ridged waveguide with short fins. Its normalized cutoff frequency is given by (4). (b) Ridged waveguide with long fins. The structure is complementary to that in Fig. 2(a) since $d_b = d'_a$. Its normalized cutoff frequency is given by (8).

The variation ΔC_1 introduced by the ridges is virtually equal to ΔC_0 since the field is quasi-uniform in the center. Thus

$$\frac{\Delta C_1}{C_1} = \frac{\Delta C_0}{C_0/2} = \frac{d'^2\pi}{2ab} = \frac{(b-d)^2\pi}{2ab} \qquad (d' \ll b, a). \quad (2)$$

Since the magnetic field is practically unperturbed by the central ridge, the shift in cutoff frequency is solely due to the change in capacitance

$$\frac{f_{c0}}{f_{cr}} = \frac{\lambda_{cr}}{\lambda_{c0}} \triangleq \sqrt{\frac{C_1 + \Delta C_1}{C_1}} = \sqrt{1 + \frac{\Delta C_1}{C_1}} \quad (3)$$

where $f_{c0} = c/\lambda_{c0}$ is the cutoff frequency of the unperturbed waveguide, and $f_{cr} = c/\lambda_{cr}$ is the cutoff frequency of the ridged waveguide. c is the speed of light. With $\lambda_{c0} = 2a$, the normalized cutoff frequency of the ridged waveguide becomes

$$\frac{b}{\lambda_{cr}} \triangleq \frac{b}{2a}\left[1 + \frac{\pi}{2}\frac{b}{a}\left(1 - \frac{d}{b}\right)^2\right]^{-1/2} \qquad (b - d \ll b, a). \quad (4)$$

This expression is satisfactory for guides with short fins.

2) Waveguides with Long Fins: In practice, waveguides with long thin ridges ($d/b \ll 1$) are of much greater interest, but, since the perturbation is large, (4) cannot be applied. Fortunately, from Marcuvitz's work on the susceptance of capacitive windows [8], a relation between the capacitances of complementary long and short fins can be derived. Fig. 2 presents two such complementary structures. Marcuvitz shows that the normalized susceptance of the short fins is

$$\frac{B_a}{Y_0} \cdot \frac{\lambda_{ca}}{b} \triangleq 2\left(\frac{\pi d'_a}{2b}\right)^2 \qquad \left(\frac{d'_a}{b} \ll 1\right) \quad (5)$$

and of the long fins is

$$\frac{B_b}{Y_0} \cdot \frac{\lambda_{cb}}{b} \triangleq 4 \ln \frac{2b}{\pi d_b} \qquad \left(\frac{d_b}{b} \ll 1\right) \quad (6)$$

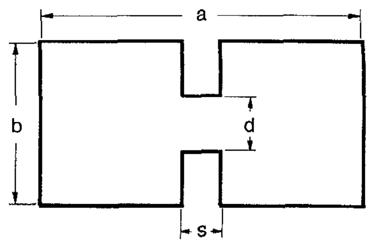

Fig. 3. Cross section of double-ridged waveguide.

where λ_{ca} and λ_{cb} are the cutoff wavelengths. When $d'_a = d_b$, the ratio of the fin capacitances is obtained by dividing (6) by (5)

$$\frac{\Delta C_b}{\Delta C_a} = \frac{(B_b/Y_0)(\lambda_{cb}/b)}{(B_a/Y_0)(\lambda_{ca}/b)} = 2\left(\frac{2b}{\pi d_b}\right)^2 \ln \frac{2b}{\pi d_b} \qquad \left(\frac{d_b}{b} \ll 1\right). \quad (7)$$

Consequently, the normalized cutoff frequency of the ridged guide with long fins can be written as

$$\frac{b}{\lambda_{cb}} \triangleq \frac{b}{2a}\left[1 + \frac{\Delta C_a}{C_1} \cdot \frac{\Delta C_b}{\Delta C_a}\right]^{-1/2}$$

$$= \frac{b}{2a}\left[1 + \frac{4}{\pi}\frac{b}{a} \ln \frac{2b}{\pi d_b}\right]^{-1/2} \qquad (d_b \ll b, a). \quad (8)$$

3) Waveguides with Fins of any Length: Both (4) and (8) can be derived from one single expression

$$\frac{b}{\lambda_{cr}} = \frac{b}{2a}\left[1 + \frac{4}{\pi}\frac{b}{a} \ln \csc \frac{\pi}{2}\frac{d}{b}\right]^{-1/2} \quad (9)$$

which transforms into (4) for $(b-d)/b \ll 1$, and into (8) for $d/b \ll 1$.

Since these expressions have been derived by assuming that the waveguide fields are only altered in the immediate vicinity of the ridges, the accuracy of (9) deteriorates with increasing ratio b/a. It is therefore necessary to correct (9) by taking second-order effects into account. The required correction cannot be determined analytically. However, it has been found empirically that if the second term in (9) is multiplied by a factor $1 + 0.2\sqrt{b/a}$, the resulting expression (10) agrees with various numerical methods [10], [15] to within one percent in the ranges $0 < b/a \leqslant 1$ and $0.01 \leqslant d/b \leqslant 1$

$$\frac{b}{\lambda_{cr}} = \frac{b}{2a}\left[1 + \frac{4}{\pi}\left(1 + 0.2\sqrt{b/a}\right)\frac{b}{a} \ln \csc \frac{\pi}{2}\frac{d}{b}\right]^{-1/2}. \quad (10)$$

Since the numerical techniques differ among themselves within this margin, the corrected perturbation formula (10) is equally reliable and accurate, and certainly more flexible than graphical design data.

4) Waveguides with Thick Ridges of any Length: Ridges of finite thickness add a second capacitance ΔC_2 to the waveguide. To a first approximation, ΔC_2 is the capacitance of parallel plates of width s and separation d (see Fig. 3)

$$\Delta C_2 \triangleq \epsilon_0 s/d. \quad (11)$$

At the same time, the width of the "unperturbed" part of the waveguide is reduced from a to $a-s$. This means that the unperturbed reference guide now possesses an aspect ratio of $b/(a-s)$, and its equivalent capacitance is reduced to

$$C_2 = \frac{1}{2}\epsilon_0 \frac{a-s}{b} \tag{12}$$

where the factor 1/2 stems from the sinusoidal distribution of the field in the guide. The relative change in capacitance is thus

$$\frac{\Delta C_2}{C_2} \triangleq \frac{2sb}{d(a-s)}. \tag{13}$$

By adding this term to the perturbation formula (10) for thin ridges, and after replacing a with $a-s$, an expression for the normalized cutoff frequency in guides with thick ridges is obtained. But, again, the term (13) does not account for second-order effects which must be evaluated empirically by comparison with numerical methods. As a result, the following corrected perturbation formula is obtained for ridged waveguide:

$$\frac{b}{\lambda_{cr}} = \frac{b}{2(a-s)}\left[1+\frac{4}{\pi}\left(1+0.2\sqrt{\frac{b}{a-s}}\right)\frac{b}{a-s}\ln\csc\frac{\pi}{2}\frac{d}{b} + \left(2.45+0.2\frac{s}{a}\right)\frac{sb}{d(a-s)}\right]^{-1/2}. \tag{14}$$

This formula agrees with numerical methods within one percent in the following ranges of parameters:

$$0.01 \leqslant \frac{d}{b} \leqslant 1$$
$$0 < \frac{b}{a} \leqslant 1$$
$$0 \leqslant \frac{s}{a} \leqslant 0.45.$$

For $s/a > 0.45$, accuracy decreases rapidly because of the mounting influence of the side walls upon the field at the ridges. At $s/a = 0.5$ (the largest ridge width of practical interest), (14) is two percent too high in the worst case: $d/b = 0.5$. Naturally, (10) and (14) are equivalent for $s = 0$.

Finally, the guided wavelength for any frequency is related to the cutoff wavelength by

$$\lambda_g = \left[1-(\lambda/\lambda_{cr})^2\right]^{-1/2} \tag{15}$$

where λ is the free-space wavelength.

B. Characteristic Impedance

The characteristic impedance of ridged waveguide is not uniquely defined. The choice of definition depends on the application of the waveguide. While Hopfer [6] adopts a definition on a voltage-to-power basis for the design of a step transformer, Cohn [5] and Chen [7] choose a voltage-to-current ratio. For predicting the interaction of fin lines (of which ridged waveguide is a special case) with semiconductor devices, Hofmann [12] and Meinel and Rembold [13] divide the voltage between the fins, calculated along a straight line, by the current in one ridge only.

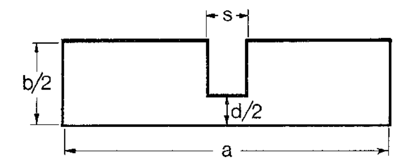

Fig. 4. Dimensions of the single-ridged waveguide that must be introduced in the expressions for the cutoff frequency and characteristic impedance.

Whatever definition of characteristic impedance may be adopted, its value depends on the frequency as follows:

$$Z_0 = Z_{0\infty}\left[1-(\lambda/\lambda_{cr})^2\right]^{-1/2} \tag{16}$$

where $Z_{0\infty}$ is the characteristic impedance for infinite frequency, and λ_{cr} is given by (14). Sharma and Hoefer [14] have derived the following formula for $Z_{0\infty}$:

$$Z_{0\infty} = \frac{120\pi^2(b/\lambda_{cr})}{\frac{b}{d}\sin\pi\frac{s}{b}\frac{b}{\lambda_{cr}} + \left[\frac{B_0}{Y_0}+\tan\frac{\pi}{2}\frac{b}{\lambda_{cr}}\left(\frac{a-s}{b}\right)\right]\cos\pi\frac{s}{b}\frac{b}{\lambda_{cr}}} \tag{17}$$

where a, b, s, and d are defined in Fig. 3.

The normalized cutoff frequency b/λ_{cr} is given by (14) and the normalized susceptance B_0/Y_0 is approximately, according to Marcuvitz [8]

$$B_0/Y_0 \triangleq (2b/\lambda_{cr})\ln\csc\frac{\pi d}{2b}. \tag{18}$$

$Z_{0\infty}$ in (17) is a voltage-to-current ratio. The voltage is the integral of the electric field taken along a straight line joining the ridges in the middle of the guide. The current is the integral of the surface current flowing in the top wall including the upper ridge.

III. Application to Single-Ridged Waveguide

All formulas derived above for double-ridged waveguide can be applied to the single-ridged waveguide with the following interpretation. In the expression for the cutoff frequency (14), b is twice the height of the single-ridged guide, and d is twice the spacing between the ridge and the bottom wall (see Fig. 4).

The same interpretation applies to the expression for characteristic impedance (17). Finally, this impedance must be divided by two to obtain the value for single-ridged waveguide.

IV. Discussion

The expression for the normalized cutoff frequency (10) of finned waveguide is presented graphically in Fig. 5 against the background of values obtained with numerical methods.

Excellent agreement exists within the full range of geometries of practical interest. A more detailed evaluation of

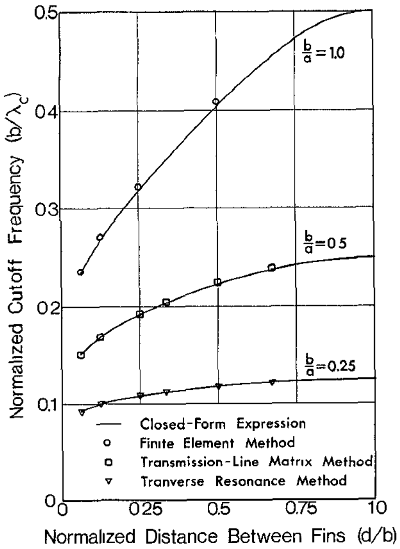

Fig. 5. Normalized cutoff frequency of finned waveguide ($s = 0$). The closed-form expression (10) is compared with numerical methods.

TABLE I
COMPARISON BETWEEN THE PROPOSED DESIGN EQUATION (10) AND VARIOUS NUMERICAL METHODS [10], [15]

d/b	Normalized Cutoff Freq. (b/λ_{cr}) of the Dominant Mode in Finned Waveguide (s = 0) b/a = 1/2				
	TRM	TLM	FEM	SDM	PROPOSED DESIGN EQUATION (10)
2/3	0.2389	0.2391	-	-	0.2379
1/2	0.2249	0.2253	0.2258	0.2249	0.2234
1/3	0.2052	0.2054	-	-	0.2039
1/4	0.1928	0.1932	0.1941	0.1935	0.1919
1/8	0.1690	0.1697	0.1710	0.1696	0.1690
1/16	0.1518	0.1522	-	0.1523	0.1525
1/32	0.1388	-	-	-	0.1400

TRM = Transverse Resonance Method
TLM = Transmission Line Matrix Method
FEM = Finite Element Method
SDM = Spectral Domain Method

this agreement can be made by studying Table I which compares the proposed design equation with results of four completely different numerical methods.

In Fig. 6, the formula for the waveguide with ridges of finite thickness (14) is compared with data published by Hopfer [6]. Actually, Fig. 6 shows the normalized cutoff wavelength λ_{cr}/a which is related to the normalized cutoff frequency b/λ_{cr} as follows:

$$\lambda_{cr}/a = \frac{1}{b/\lambda_{cr}} \cdot \frac{b}{a}. \qquad (19)$$

Agreement is better than one percent for s/a values up to 0.45 in the most unfavorable case of $d/b = 0.5$. Higher values of s/a are seldom of practical interest.

The closed-form design expressions (10) and (14) have the following advantages over all other methods published

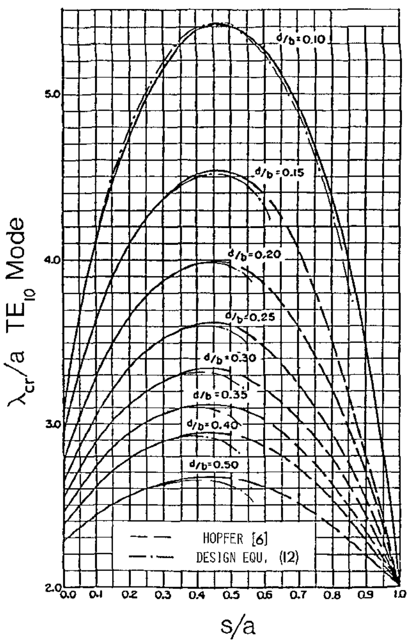

Fig. 6. Normalized cutoff wavelength of ridged waveguide. The closed-form expression (14) is compared with values published by Hopfer [6].

to date: they are easily and quickly evaluated by hand or programmed on a computer or calculator; they can be used for design as well as for analysis since any parameter can be isolated on one side of the equation; they can be easily differentiated with respect to any parameter to determine sensitivity to tolerances; they are less subject to computation error than all other methods; and they are ideally suited for computer aided design and manufacturing of circuits, in particular when dimensions vary continuously.

The accuracy of the expression for the characteristic impedance (17) is the same as that of (14) but its usefulness depends on the adequacy of its definition in a particular situation.

V. CONCLUSION

Closed-form expressions for the cutoff frequency and the characteristic impedance of ridged waveguide have been developed. Initially, first-order expressions have been derived using perturbation theory and Marcuvitz' formulas for the susceptance of capacitive windows. Then, corrective terms were introduced empirically to account for second-order effects. The resulting expressions agree with numerical methods within one percent for all geometrical parameters of practical interest. Because of their simplicity, these new expressions considerably simplify the design of ridged waveguides without any concession in accuracy. They have the advantage of great flexibility, can be differentiated directly for tolerance analysis, and may be easily programmed for computer aided design and manufacturing.

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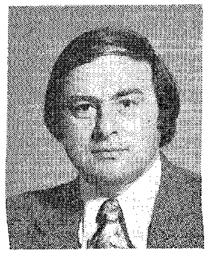

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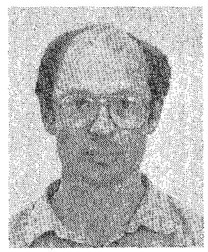

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A Compact Broad-Band Multifunction ECM MIC Module

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***Abstract*—A development effort is described that yielded a compact broad-band ECM module using soft and hard substrate material employing microstrip, slotline, and coplanar line. Integrated functions include coupling, limiting, upconversion, downconversion, broad-band amplification, amplitude modulation, switching, gating, and stable frequency generation. A high-level frequency converter with a +28-dBm intercept point resulted in high dynamic range, spurious-free operation (−45 dBc). Extremely flat amplification with low-current drain is achieved with distributed and cascode FET amplifiers at S–C and X-bands.**

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I. Introduction

REDUCING THE size, weight, and power consumption of modern electronic systems requires compact, efficient, plug-in, multifunction modules. Key parameters are broad bandwidth, flat frequency response, low power consumption, high speed, high dynamic range, and low spurious signal generation. This paper describes details of the microwave substrate materials and layout to miniaturize

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